LIMITS

Math 130 - Essentials of Calculus

10 February 2021

Concept of a Limit

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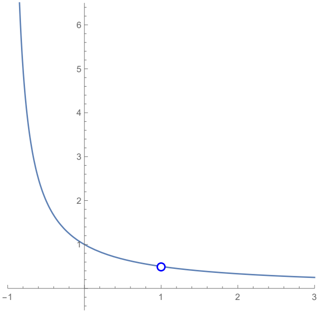

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What is happening to the value of $f(x)$ as the value of x is getting closer to 1? On the left side, values are approaching 1 from the left, and values are approaching 1 from the right in the second pair of columns.

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So it appears the values are approaching 0.5. We say $\lim_{x \to 1} \dfrac{x-1}{x^2-1} = 0.5$.

Definition of a Limit

Definition

We write

$$\lim_{x \to a} f(x) = L$$

and say "the limit of $f(x)$, as x approaches a, equals L"
if the values of $f(x)$ approach L as the values of x approach a (but are not equal to a).

Estimating a Limit

Example

Use a table of values to estimate the value of the limit

$$\lim_{h \to 0} \frac{\ln(h+1)}{h}.$$

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Theorem (Limit Laws)

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6. $\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$ *for positive integers n*

Using the Limit Laws

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6. $\lim_{x \to 5} \dfrac{3w^2 + 1}{w}$

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6. $\lim_{x \to 5} \frac{3w^2 + 1}{w}$
7. $\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$

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7. $\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$ Something's not right here...

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Examples of Continuous Functions

The following types of functions are continuous on their domains. This means, as long we're taking the limit to a value in the domain of the function, we can just plug the number into the function.

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- root functions
- exponential functions
- logarithmic functions
- trigonometric functions

Using Continuity to Evaluate a Limit

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2. *Compute* $\lim\limits_{x \to 4} f(x)$.
3. *Compute* $\lim\limits_{x \to 2} f(x)$.

Now You Try It!

Example

Consider the function $f(x) = \dfrac{x^2 - 2x - 3}{x + 1}$.

1. *What is the domain of* f*?*
2. *Compute* $\lim\limits_{x \to 1} f(x)$.
3. *Compute* $\lim\limits_{x \to -1} f(x)$.